SOME RECENT MATHEMATICAL BOOKS.

Euclid. Part 1, Books I. and II. By H. S. Hall and F. H. Stephens. (Macmillan, 1887.)
Algebraical Exercises. By H. S. Hall and S. R. Knight. (Macmillan, 1887.)
Key to Todhunter's Mensuration By Rev. L. McCarthy. (Macmillan, 1886.)
Explanatory Arithmetic. By G. E. Spickernell. Third Edition. (Simpkin, 1887.)
Plane and Spherical Trigonometry. By H. B. Goodwin. (Longmans, 1886.)
Spherical Trigonometry. Part 2. By W. J. McClelland and T. Preston. (Macmillan, 1886.)
Solid Geometry: Solutions. By P. Frost. (Macmillan, 1887.)
Elementary Statics. By J. Greaves. (Macmillan, 1886.)
Differential Calculus. By B. Williamson. Sixth Edition. (Longmans, 1887.)
Differential Calculus. By J. Edwards. (Macmillan, 1886.)
Algebra. By Oliver, Wait, and Jones. (Ithaca: Finch, 1887.)
Practical Solid Geometry. By W. G. Ross. (Cassell, 1887.)

IF former periods of the world's history were characterized as the "Stone," "Bronze," and "Iron" Ages, the present epoch might well be entitled the "Book" Age. Amidst the flood of literature of all sorts which daily pours out of the jaws of our printing leviathans, didactic mathematics certainly claims its due proportion. The number of Algebras, Euclids, Arithmetics, and Trigonometries which appear, with the "rough ways made smooth and the crooked ways straight," make us regard the modern student with a mixture of envy and pity—envy at the possession of such broad highways to knowledge as we never dreamt of, and pity at the difficulty he must experience in choosing out of such a multitude. We cannot, moreover, help fancying that this plethora of books is not entirely without compensating disadvantages, for the very ease and tranquillity which which the student glides through the cleared forest, make him careless and inattentive of the land-marks and salient features that were so carefully noted by his more self-reliant, if less luxuriously-equipped, predecessor.

Of the twelve books on our table, we shall begin by noticing the most elementary. Among these is an instalment comprising Books I. and II. of "Euclid," by Messrs. Hall and Stevens, of Clifton College. For these revolutionary days it is remarkably orthodox; but certain changes have been introduced, very wisely as we think, where Euclid's enunciations were confusing, or the proofs were not sufficiently comprehensive, as in Propositions 8 and 26, where the identical equality of the two triangles is not usually emphasized. The authors, in their preface, enter very fully into the reasons which decided them to avoid the use of symbols at first, and also to preserve "the formal, if somewhat cumbrous, methods of Euclid," and with these reasons we in the main agree. If, however, for "a large majority of students 'Euclid' is intended to serve, not so much as a first lesson in mathematical reasoning, as the model of formal and rigid argument which most conduces to accurate and orderly thought in any field of study," we should welcome a book of geometry brought out for the use of those whose natural mathematical growth is stunted, and taste warped, by a too strict adherence to the cumbersome and often involved style of the ordinary text.

We admit the dilemma for those who wish to make "Euclid" serve the double purpose of an introduction to logic as well as geometry, but, at the same time, we are unable to see why, even for the logicians, *some* at least of the advantages of the German method cannot be introduced, such as the use of a, b, c for the sides, and α, β, γ for the angles of a triangle. A great deal of confusion arises from the use of *three* letters for an angle and *two* for a side, and the change to the simpler method would not only clear the student's mental vision, but leave the logic unimpaired.

One of the best examples of the defects arising from a rigid adhesion to the formal text is in Book I., Proposition 13. We have found many to whom this proposition in its existing form is one of the most repulsive in the book, and it has been almost touching to witness the joy evinced by a dullard on his first realizing that the obtuse angle was just as much in excess, as the acute angle was in defect of a right angle—a fact which the ordinary proof completely disguises. We do not think the alternative proof to Proposition 47 is likely to meet with much favour, and since the authors do not altogether discard alternative proofs, we should have preferred, instead, the neat alternative to Proposition 48 given in Casey.

For the purpose for which it is designed we do not hesitate to recommend the book. It is excellently printed, the construction lines being very properly faint, and the figures in all cases clearly drawn. The exercises, additional theorems, and hints to solution are also unusually well arranged, and will be truly welcome to the student who intends to go in for mathematics, as well as train his mind into logical habits.

"Algebraical Exercises," by Messrs. Hall and Knight, is, we presume, intended to be a companion and supplement to their excellent little "Elementary Algebra," which has met with such a generally good reception. While these exercises will, no doubt, be of considerable use, we think they might be improved, and rendered more widely serviceable, (1) if Part 1, devoted to the earliest rules, were extended beyond a meagre six pages—a range altogether out of proportion to what follows; and (2) if the exercises were more *gently* graduated in Part 2. For example, linear equations in three unknowns are introduced *per saltum* as early as p. 10, and all the papers after p. 6 strike us as being a good deal harder than those encountered by the Army Preliminary candidate, types of which are given on pp. 146–47. We also regret to find in the first ninety-seven pages an almost entire absence of book-work questions—a defect which is only partially made up for in the capital selection of typical public examination papers which follow. Example-grinding is no doubt an essential art, but in algebra, especially, the early parts tend to become purely mechanical, unless *real thinking* is encouraged and stimulated by rational questions on the processes employed. In a subsequent edition a few more recent University and Army papers would form a welcome addition.

Mensuration is represented by a key to Todhunter's small treatise, by the Rev. Lawrence McCarthy, of St. Peter's College, Agra. As a rule we have no great liking for keys, but if there is any country where such a key might be used with advantage, it is India. Possibly this is what prompted Prof. McCarthy to perform a labour which might fitly be termed an intellectual treadmill. We have always looked upon the "Mensuration" itself as Todhunter's least valuable work. It is full of long-winded rules which are never learnt, and the use of algebraic symbols by which all such rules could at once be rendered visible to the eye, is most curiously, and, as we think, unreasonably, avoided. The same spirit characterizes the key. Everything is worked out at full length with needless repetitions of figures, especially the oft-recurring 3·1416, which we have counted no less than twenty-nine and thirty-four times respectively on two pages chosen at random. Surely the symbol π might have been substituted with advantage here. Barring such defects, the work appears to have been well done, and will doubtless be of use, more especially to those who are unable to get tutorial assistance.

An "Explanatory Arithmetic," by Mr. G. E. Spickernell, which has reached a third edition, does not strike us as any improvement on existing treatises. Brevity, which is one of the points aimed at, is certainly secured, but at the expense of both elegance and lucidity. The rules read like excised telegrams, and are liable to be misconstrued in much the same way. Thus, to take an example of the telegram purporting to explain compound subtraction: "Take *like from like;* and whenever it is necessary, in order to make subtraction possible"; and a longer one for the subtraction of fractions runs thus: "Reduce minuend and subtrahend to equivalent fractions having their least common denominator; and then, having like parts of integers, take less number from greater, and write in figures, under remaining parts, their name." Similar highly cacophonous and ambiguous paragraphs are to be found scattered through the book, and give one the impression that they will frequently necessitate as much explanation as the principles they are intended to embody. Occasionally the author employs a definition which is palpably partial, thus: "When an integer or whole thing is divided into a number of *equal* parts, those parts are called fractions." The entire book strikes us as being of the empirical cramming style, as opposed to the rational and scientific style so well exemplified in Brook Smith's treatise and the smaller one by Lock, in which rules are avoided as much as possible. On the other hand, it contains copious and very well assorted collections of examples and examination papers, with answers which can be readily removed from it if desired. These might be used with advantage, and the teacher, if a good one, could translate and expand the telegrams into a more rational and elegant form, or, still better, do without them.

Trigonometry, plane and spherical, is represented by two books—one, comprising the elements of both subjects, chiefly for the use of junior naval officers, by H. B. Goodwin, Naval Instructor; while the other is Part 2 of a "Treatise on Higher Spherical Trigonometry and Geometry," by Messrs. McClelland and Preston, of Dublin. Mr. Goodwin's work is intended to give, in one volume, the course usually required for an acting sub-lieutenant—which heretofore he has had to pick out of a variety of elementary works—and appears to admirably fulfil its author's intention. It is marked by simplicity of treatment, the avoidance of cumbrous rules, those bugbears of our ancient text-books, and a separation of the subject into distinct parts, each of which is complete in itself.

Messrs. McClelland and Preston's book is a new departure, in so far as, with the exception of the well-known treatise of Mulcahy, it is the first time that spherical geometry, as distinct from trigonometry, has been seriously put into text-book form. The authors are to be congratulated on their bold, clear, and systematic treatment of this too-much-neglected and really useful branch of mathematics. The work throughout is characterized by lucidity and originality of treatment, and is subdivided into complete chapters. Spherical and stereographic projection are also carefully explained, and their power as methods amply exhibited.

We cannot help thinking that both spherical trigonometry and geometry are far too much neglected in our educational curricula. In consequence, it is astonishing what errors are committed when even the simplest properties of a spherical surface are in question. The curvature of the earth is realized by few, and some who ought to know better have not yet grasped the fact that the latitude of Cairo approximately bisects the area of the northern hemisphere. Nothing but polar projections, or, better still, globes themselves, will ever correct the false impressions which we get from that terrible flat-ruled distortion entitled "Mercator's projection," from which all approach to curvature has been so carefully extracted.

The theory of the trade-winds, moreover, which has survived up to date in some of the text-books, takes no account of the shortening of the radius in considering the gain in eastward motion by the transference of the air at relative rest on the equator to higher latitudes. Thus, according to Loomis's "Meteorology," the gain is simply found by subtracting the linear velocity at the higher latitude from that at the equator; whereas when the shortened radius is considered, it amounts, in latitude 60°, to 1554 instead of 518 miles per hour, and at the Pole itself to ∞ instead of 1036 miles per hour. These are only a few of the most patent errors which arise from a neglect of spherical principles, and might be multiplied almost indefinitely.

"Hints for the Solution of Problems in Solid Geometry," by Dr. Percival Frost, is a book which cannot fail to be of great value to the student of this difficult but important branch of mathematics. Mathematical solutions have little analogy to, and, except in elementary works, none of the disadvantages of, classical cribs. In the present case, the execution of Dr. Frost's truly laborious work has been attended by an unexpected advantage, in leading to the discovery of certain errors and omissions in the statement of the problems themselves, which might otherwise have escaped notice. We heartily welcome Dr. Frost's hints, and trust they may receive the attention they so fully deserve.

While some branches of elementary mathematics are already in danger of being congested by a plethora of text-books, statics and dynamics seem to us to still present an open field for the writers of really able didactic treatises. For those who have reached the Elysian fields of the calculus, no better book can be recommended than Prof. Minchin's admirable work; while for those who have not yet reached that stage, and perhaps never intend to, the "Elementary Statics" by Mr. John Greaves, which is written much on the same lines, can be unreservedly recommended. The science of statics, like everything else, has been obliged to *move* with the times, and Mr. Greaves, following the modern views, prefers to consider it as merely a particular case of the science of dynamics, and to base it upon the laws of motion. Thus, instead of the familiar proof of the parallelogram of forces on the principle of transmissibility of force, he deduces it solely by the aid of the parallelogram of velocities, from which, together with the third law of motion, the conditions of equilibrium are obtained more readily, and, in the author's opinion, more clearly, than usual. This expectation can only be tested by actual experience. Meanwhile we would recommend that, in a reprint of the book, the more salient propositions and results should be rendered more emphatic and conspicuous by being placed either in italics or large type. In their present form and position the plums are too disguised to be readily picked out. The work is characterized by thoroughness, and by a large number of worked-out examples illustrated by excellent figures, the material lines being very properly distinguished from the geometrical and force lines by thicker type. The free use made by the author of the purely geometrical method for solving some statical problems is elegant, but occasionally leads to a neglect of the statical or primary limitations under which they are stated. An example of this occurs on p. 86, where it is required to find the greatest inclination to the horizon at which a uniform rod can rest partly within and *partly without* a fixed smooth hemispherical bowl. The condition assumed for the maximum inclination leaves *no part* of the rod outside the bowl, which clearly violates the latter part of the question.

Machines are deferred to the last chapter in the book, presumably because some of the principles, such as virtual work, are dealt with in preceding chapters; but we think they might be advantageously introduced, at all events in a preliminary way, much earlier, since their consideration not only enlivens the otherwise dry discussion of abstract principles, but gives concrete expression to their reception. We are glad to see that the merits of this excellent little book are recognized by the authorities of the Mason Science College, who recommend it for one of their courses.

At the threshold of the higher mathematics we find two books on "Differential Calculus," which, though rivals, will, we trust, often be found in company, since each possesses certain merits and characteristics which distinguish it from the other. One is the well-known and excellent treatise by Prof. Benjamin Williamson, F.R.S., which has now reached a sixth edition. In this edition, besides careful revision, a short discussion is added on the elementary properties of solid and spherical harmonics, which are so frequently employed in the higher developments of electrical and optical theories. As a former edition of the book has been fully noticed in NATURE, we need only indorse the opinion then put forward, that it is one of the best treatises on the subject in our language. The other work, by Mr. J. Edwards, formerly Fellow of Sidney-Sussex College, Cambridge, is very different in style, and more elementary, in so far as it is, according to the author's design, "unencumbered by such parts of the subject as are not usually read in Colleges and schools." Compared with Prof. Williamson's treatise, it is distinctly more geometrical in method, and in this and some other points, such as large type, beautifully-drawn figures, an unusually full and systematic account of curve-tracing and the properties of curves, which, contrary to the usual custom, *precede* maxima and minima, it is more suited to the wants of the average student as a preliminary course of reading. Some of the geometrical illustrations, such as those of the compressed form of Taylor's theorem, $\phi(x+h) = \phi(x) + h\phi'(x+\theta h)$, and $Dz = \frac{dz}{dx}dx + \frac{dz}{dy}dy$, are very elegant, and help to keep alive the real meaning of differential symbols, which a too exclusive attention to algebraic analysis tends to annihilate. Symmetry and brevity have both been evidently studied, and a good example of this may be seen on p. 271, where the radius of curvature for an implicit function of x and y is deduced. If this be compared with the analogous method on p. 290 of Williamson's book, the difference in the style will be manifest. Regarding the two books together, we should advise a student to begin with Edwards, and then proceed with Williamson. Nothing in the former work need be omitted at a first reading, after which he may plunge fearlessly into the more complete and analytical treatise of the Dublin Professor.

Two books remain to be noticed, which lie somewhat outside the ordinary run of didactic works. One is an Algebra by Profs. Oliver, Wait, and Jones, of Cornell University, U.S. This, though originally intended as a text-book for their own students, seems, in the course of construction, to have developed into a work which, while it might be found really useful as a book of reference to teachers and the rare youth who *cultivates* mathematics, is quite unsuited to the ordinary student.

In some respects it appears to be an effort to regard algebra from the modern point of view as the science of finite operations, and to present it in the form of "a stepping-stone to the higher analysis," and there is much that is commendable from this point of view in the exposition of incommensurables, limits, imaginaries, derivatives, and graphic representation of equations, as also in the introduction of some fresh symbols, such as the Gaussian sign of congruence, $\equiv$, and $\rightarrow$ $\leftarrow$ for smaller than and greater than in the sense of size only. The use of the signs $+ -$ as left-hand indices to indicate absolutely positive and negative quantities is also an improvement, and renders it easier to deal with negative and directional quantities. For English didactic purposes, however, this book will be chiefly useful as one of reference for the teacher.

In conclusion, we must not omit to draw attention to a very handy little manual of "Practical Solid Geometry," by Major Gordon Ross, of the Royal Military Academy, Woolwich, which is particularly adapted to military students. The method of orthographic projection by

"vertical indices" receives special attention, and a section on defilade—a subject not much studied in this country—cannot fail to be of use and interest to those who study the science of war.
E. D. A.

OUR BOOK SHELF.

Examples in Physics. By D. E. Jones, B.Sc., Lecturer in Physics at the University College of Wales, Aberystwyth. (London and New York: Macmillan and Co., 1888.)

SO many books have been written having titles similar to if not identical with that quoted above, the only object of which seems to have been to enable students to pass certain examinations with the minimum of knowledge, that it is a comfort to turn to one against which no such charge can be made. Mr. Jones's "Examples in Physics" has not been written "up to" any Syllabus, but the author has made use of portions of the manuscript in teaching classes of students taking the intermediate science and preliminary scientific courses of the London University, and he believes it will be found useful for students who are preparing for these examinations. There can be no doubt that the book will be of great assistance in this way, owing to the large number of examples and the excellent way in which they have been graduated. In addition to the examples, of which there are more than a thousand, with occasional hints for their solution, there are short explanatory chapters and paragraphs where experience has shown that they are needed. Thus, at the beginning, the C.G.S. units are thoroughly explained, as is the method of passing from one system of units to another by means of dimensional equations. Those approximate relations which are most often made use of are shown to be true, and examples illustrating the advantage of employing them are worked out. The method of using logarithms is explained, and both on pp. 19 and 21 the reader is told that there is a table of four-place logarithms at the end of the book. There is a page on which natural sines and tangents are given to three places, but not a vestige of a logarithm is to be found.

In the chapters on dynamics, hydro-statics, heat, light, sound, electricity, and magnetism, chapters which consist essentially of examples, there are clearly-written paragraphs explaining those points that do not generally seem to be grasped by students. The answers to the questions are given at the end.

The general arrangement of the book is particularly happy ; it is clearly the work of a teacher whose object is to increase the real knowledge of his students, and not merely to drive them through the ordeal of an examination.

The Constants of Nature. By Frank W. Clarke. Part I. New Edition. (Washington : Published by the Smithsonian Institution, 1888.)

THIS volume consists of a series of tables of specific gravities of solids and liquids, and differs from the older edition in two respects. In the first place, the tables have been revised and greatly enlarged ; and, secondly, melting and boiling points have been omitted, on the ground that they are already supplied by the two volumes by Prof. Carnelley, which are specially devoted to those data. How much the tables have been enlarged may be gathered from the fact that the older edition, with a later supplement, only gave 2963 substances, whereas there are now no less than 5227 distinct substances mentioned, and 14,465 separate determinations. As the author remarks, this increase is a noteworthy indication of chemical activity.

The tables are only intended to be complete as far as artificial substances of definite constitution are concerned, but, in addition to these, many minerals find places. For each substance, the formula, specific gravities, and authorities are stated. The elements take the first place, and these are arranged in order of densities. Then follow inorganic fluorides, chlorides, bromides, iodides, oxides, sulphides, &c., the various groups of organic bodies coming last. There is a very complete index to the names of substances, without which, of course, the book would be far from complete.

The author is to be congratulated on the successful completion of an undertaking entailing such a vast amount of patient labour.

LETTERS TO THE EDITOR.

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Gresham College.

I TRUST you will allow me space for a short note upon Prof. Lankester's article in last week's issue of NATURE (p. 1), under the above heading. I have no intention of entering into a controversy with Prof. Lankester upon matters of opinion, but I am desirous of correcting statements, made by him in that article, which are inaccurate. In point of fact, the article is based entirely upon a misapprehension as to the purpose and function of the London Society for the Extension of University Teaching, and its position with regard to Gresham College. Prof. Lankester speaks of it as a Lecture Society, and refers to the "innumerable short courses" of lectures given in different halls, Vestry Halls, and others in London. He is evidently here confusing two distinct things. The short courses in large halls are given under the People's Lectures Scheme, which is an entirely different matter, and in connection with which we never use the word "University." The courses of lectures and classes carried on by the London Society and the Universities Joint Board involve a systematic course of work extending over a full term of ten or twelve weeks. It is a part of the same great movement which is carried on by the University of Cambridge (not by a Lecture Society), and more recently also by the University of Oxford, in different parts of the country.

Those who have had an insight into the methods and the result of the working of the University Extension movement have been struck with the excellence and thoroughness of the work done. Sir James Paget, in his annual address to students at the Mansion House, last February, spoke as follows with regard to some of the syllabuses :—

"As I looked through the syllabuses of such subjects as I can estimate, I could see that the amount of teaching in each of them is enough for a good beginning for some who may intend to make that subject a chief study for their lives, and enough to form an important part in the teaching of anyone who wishes to be in the fairest sense generally well educated."

In fact, the principles which underlie the University Extension movement cannot but meet with the approval of all interested in higher education. These principles are, "first, that the amplest facilities for the best kind of higher teaching, such as the Universities provide, should be brought within the reach of the great mass of the people by means of courses of instruction, given locally at convenient times and places ; and, secondly, that a teaching system, as opposed to a mere examination system, is required to meet the educational needs of the time."

In his speech at Gresham College, Mr. Goschen made no claim that "Gresham's money should be assigned to the support of the lecturers of the Society." What he did point out was that the design of Sir Thomas Gresham was to establish, in the heart of London, University teaching for busy people engaged in the City, and that the aim of the University Extension movement, whether in London or the country, was in spirit the same.

Prof. Lankester charges this Society with making an objectionable use of the word "University" in order to gain financial support. He says, "the implication is that the teaching is such as is given at the Universities, and it is an entirely false implica-